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目录

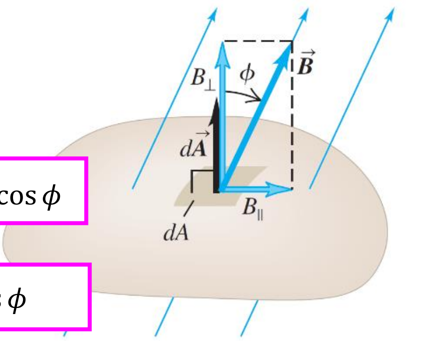
Magnetic Flux 磁通量

Magnetic Flux 磁通量

$$\Phi_m = \int_A \vec{B} \cdot d\vec{A} = \int_A B\, dA \cos\phi$$

$\vec{B}$ is uniform, A is flat → $\Phi_m = \vec{B} \cdot \vec{A} = BA\cos\phi$

Coil has N turns → $\Phi_m = NBA\cos\phi$



Faraday's Law 法拉第电磁感应定律

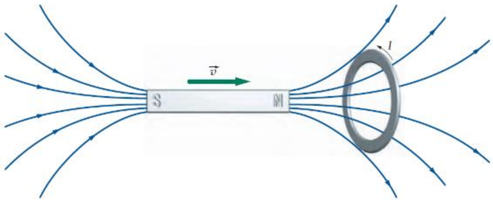
Inducted EMF 感生电动势

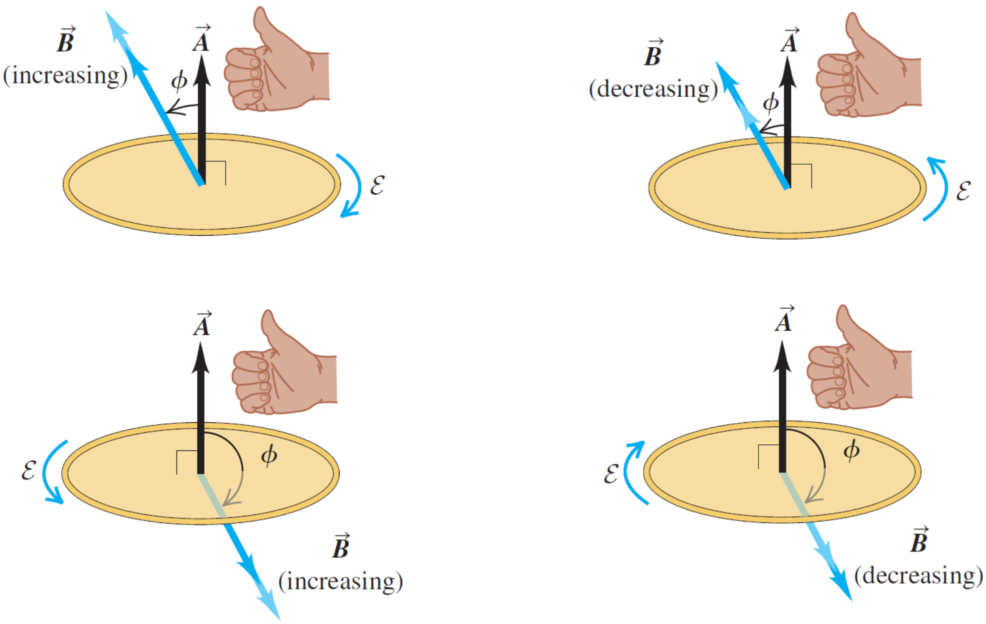
$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

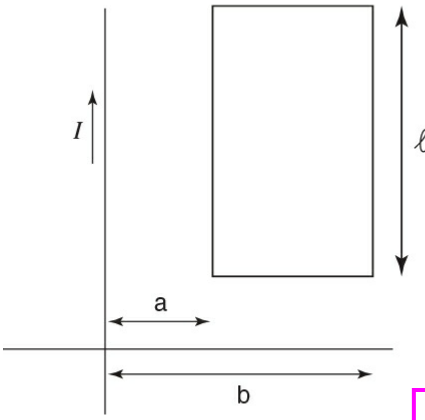
Lenz's Law 楞次定律

- The induced emf is in such a direction as to oppose, or tend to oppose, the change that produces it.
- 感生电流的磁场方向与磁通量的变化方向相反。或者感生电动势的方向总是反抗引起感生电动势的原因。





Faraday's Law: An infinitely long wire lies on the y-axis and carries a current given by $I(t) = ct^2$, where c is a constant with units of A/s^2, as shown in the right figure. Calculate the EMF experienced by the loop of wire shown as a function of time.



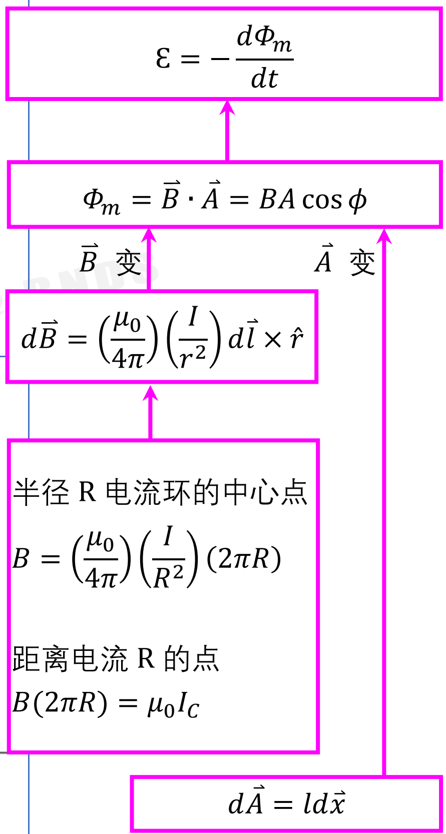
思路：1. $\mathcal{E} = -\frac{d\Phi_m}{dt}$ 求出Φ_m的表达式并且微分即可

2. $\Phi_m = \int_A \vec{B}\cdot d\vec{A}$需要求出 B 和 A 的关系

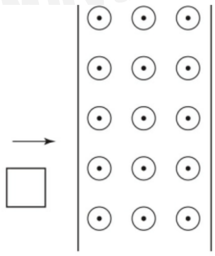
3. 安培环路定理$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C \xrightarrow{\text{距离电流}x\text{处}} B(2\pi x) = \mu_0 I$,

$dA = l dx, \Rightarrow \Phi_m = \int_A \vec{B}\cdot d\vec{A} = \int_{x=a}^{x=b} \left(\frac{\mu_0 I}{2\pi x}\right)(l dx) = \frac{\mu_0 I l}{2\pi}\ln\frac{b}{a}$

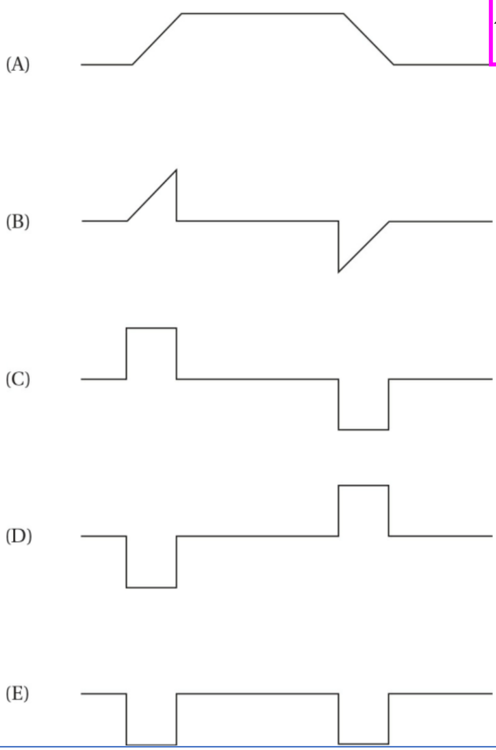
4. $|\mathcal{E}| = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}\left(\frac{\mu_0 I l}{2\pi}\ln\frac{b}{a}\right) = \left(\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}\right)\frac{dI}{dt} = \left(\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}\right)(2ct)$



Lenz's Law: An external force pushes a square loop of wire through a region of constant magnetic field as shown in the right figure.



1. Which of the following is a plot of the magnetic flux as a function of the position of the loop?
2. which of the plots shows the induced current in the loop (with clockwise current designated as positive)?

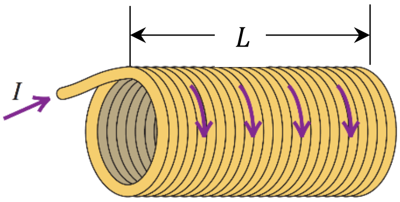


答案： 1. A， 2.C

Inductance 电感

Self-Inductance 自感

例如 solenoid:



$B=\mu_0 nI,\qquad n=\dfrac{N}{l}$ (N扎数，l长度)

$\phi_m = NBA\cos\varphi$

$=(nL)(\mu_0 nI)(\pi R^2)\cos 0 = \mu_0 n^2\,\pi R^2 l\, I$

定义：

$V=\pi R^2 l$

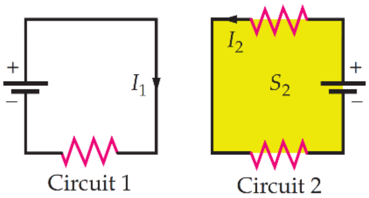
$\phi_m = LI \quad \Rightarrow L=\dfrac{\phi_m}{I}=\mu_0 n^2 V \quad L{:}\,Self-Inductance$

Mutual-Inductance 互感

$\phi_{12}=M_{12}I_1$

定义：

M_{12}: 1 对于 2 造成的互感



Energy Stored in an Inductor 电感储能

$U=\dfrac{1}{2}LI^2$

对比记忆：

电容：$U=\dfrac{1}{2}QV=\dfrac{1}{2}\dfrac{Q^2}{C}=\dfrac{1}{2}CV^2$

电阻：$P=IV=\dfrac{V^2}{R}=I^2R$

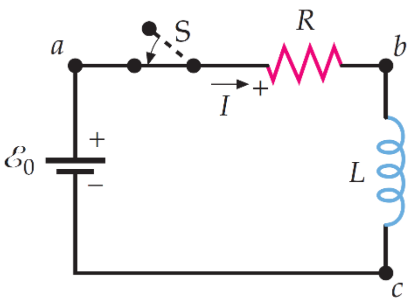
R-L 电路

Charging

$$\begin{cases}\mathcal{E}=IR+L\dfrac{dI}{dt}\\ I(t=0)=0\\ I(t=\infty)=\mathcal{E}/R\end{cases}$$

$I(t)=I_\infty\left(1-e^{\frac{-Rt}{L}}\right)$

$V(t)=L\dfrac{dI}{dt}=\mathcal{E}e^{\frac{-Rt}{L}}$

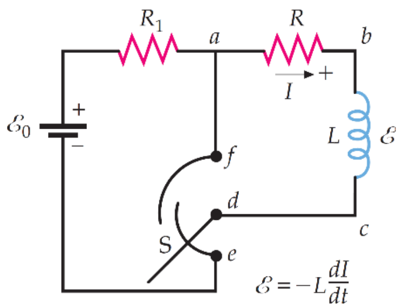


Discharging

$$\begin{cases}IR+L\dfrac{dI}{dt}=0\\ I(t=0){:}\text{已知}\\ I(t=\infty)=0\end{cases}$$

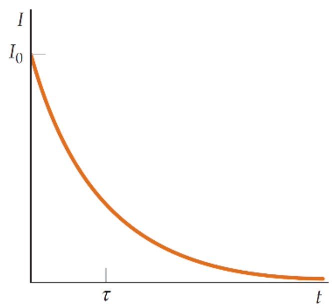
$I(t)=I_0e^{\frac{-Rt}{L}}$

$V(t)=L\dfrac{dI}{dt}=(-I_0R)e^{\frac{-Rt}{L}}$



Time Constant

$\tau=L/R$



Maxwell's Equations 麦克斯韦方程组

变化的电场产生磁场 Ampère's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0(I + I_d) \quad where\ I_d = \varepsilon_0 \frac{d\Phi_e}{dt}$$

变化的磁场产生电场 Faraday's Law

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d\Phi_m}{dt} \qquad 或\ \ \varepsilon = -\frac{d\Phi_m}{dt}$$

封闭曲面包围的电场(电通量)Gauss's Law

$$\oint_S E_n dA = \frac{Q_{in}}{\varepsilon_0} \qquad 当\Phi均匀时:\ \ \Phi_{net} = E_n A = \frac{Q_{in}}{\varepsilon_0}$$

封闭曲面包围的磁场(磁通量)Gauss's Law for Magnetism

$$\oint_S B_n dA = 0$$

麦克斯韦方程组 $\begin{cases}\text{变化的电场产生磁场}\\ \text{变化的磁场产生电场}\end{cases} \Rightarrow$ 电场磁场交互变化产生电磁波

AP Physics C- Class EM Homework 7

Chinese Name:__________ English Name:__________ Section No.__________

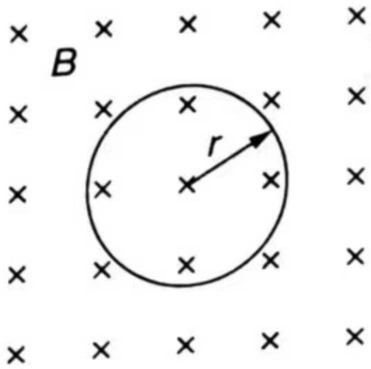
Answer all of the following questions:

1. A student wants to construct an inductor of a given inductance using copper wire and a plastic tube. If a sufficient supply of copper wire is available, the student will also need a

A) meterstick only
B) secondary coil and a meterstick
C) resistor of known resistance and a meterstick
D) voltmeter and a meterstick
E) voltmeter and a DC power supply

$L=\mu_0 n^2 V$
1. 答案 A

2. A single loop of wire is at rest in a magnetic field that is directed into the page, as shown in the figure. The loop has a radius of r and a resistance of R. The magnetic field has a magnitude B that changes as a function of time t according to the equation $B=\alpha t+\beta t^2$, where α and β are both positive constants in units of T/s and T/s^2, respectively. Determine the current in the loop as a function of time t.

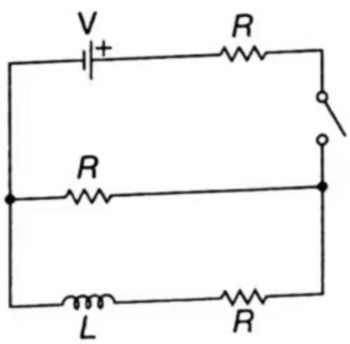


半径 r 电流环的中心点

$$B=\left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{r^2}\right)(2\pi r)=\frac{\mu_0 I}{2r}$$

$$\Rightarrow I=\frac{2r}{\mu_0 B}=\frac{2r}{\mu_0(\alpha t+\beta t^2)}$$

3. A circuit is constructed with the battery, inductor, switch, and three identical resistors shown. The switch is initially open and then is closed. Determine



a) The energy stored in the inductor after the switch has been closed for a very long time.

t=∞时 L 处于短路状态

$I=V/2R$

$$U=\frac{1}{2}LI^2=\frac{1}{2}L\left(\frac{V}{2R}\right)^2=\frac{LV^2}{8R^2}$$

b) The current provided by the battery immediately after the switch is closed.

t=0 时 L 处于开路状态

$I(\mathrm{t}=0)=V/2R$

4. A long, straight wire carrying a constant current creates a magnetic field around it. The magnetic field strength is measured with a magnetic field sensor at varying distances from the wire while the current is kept constant. Which of the following indicates a graph that will have a constant positive slope and the Maxwell equation most closely related to the graph?

A) A graph of the magnitude of the magnetic field as a function of the current. This is related to Faraday's law. *Faraday's Law:* $\varepsilon = -\frac{d\Phi_m}{dt} \Rightarrow IR = -\frac{dB}{dt}A\cos 0 \Rightarrow I$不变，A错误

B) A graph of the magnitude of the magnetic field as a function of distance from the center of the wire. This is related to Faraday's law.

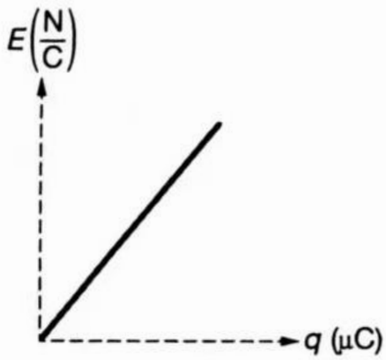
C) A graph of the magnitude of the magnetic field as a function of distance from the center of the wire. This is related to Ampere's law. *Ampere's Law:* $B(2\pi R) = \mu_0 I_C \Rightarrow B$不与$R$呈正比，错误。

D) A graph of the magnitude of the magnetic field as a function of the inverse of the distance from the center of the wire. This is related to Faraday's law.

E) A graph of the magnitude of the magnetic field as a function of the inverse of the distance from the center of the wire. This is related to Ampere's law.

Ampere's Law: $B(2\pi R) = \mu_0 I_C \Rightarrow B$与$1/R$呈正比：E正确

5. An experiment is run on a charged sphere. Students measure the charge q on the sphere and the magnitude of the electric field E a constant distance from the center of the sphere. The data are shown in the graph. Which of the following indicates a physical constant that can be determined from the graph and indicates the Maxwell equation associated with this graph?



A) ε_0; Gauss's law

B) ε_0; Ampere's law

C) μ_0; Gauss's law

D) μ_0; Ampere's law

E) μ_0; Faraday's law

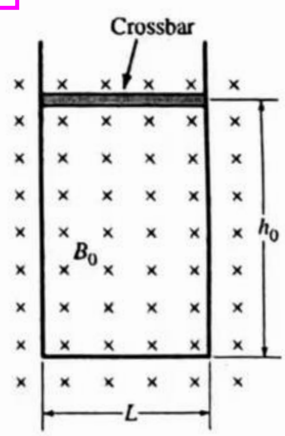
$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}\hat{r}$$

Gauss's Law **电通量** $\Phi_{net} = \oint_S \vec{E}\cdot\vec{n}dA$

Ampere's Law **磁通量** $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C$

ε_0:通过电通量Φ_{net}可得⇒答案(A)

6. A closed loop is made of a U-shaped metal wire of negligible resistance and a movable metal crossbar of resistance R. The crossbar has mass m and length L. It is initially located a distance h_0 from the other end of the loop. The loop is placed vertically in a uniform horizontal magnetic field of magnitude B_0 in the direction shown in the figure. Express all algebraic answers to the questions below in terms of $B_0, L, m, h_0, R,$ and fundamental constants, as appropriate.



a. Determine the magnitude of the magnetic flux through the loop when the crossbar is in the position shown.

$$\Phi_m = \vec{B}\cdot\vec{A} = B_0(h_0L)$$

b. The crossbar is released from rest and slides with negligible friction down the U-shaped wire without losing electrical contact.
On the figure below, indicate the direction of the current in the crossbar as it falls

Justify your answer.

The $\overrightarrow{B}$ of the closed area is decreasing, so the inducted Mag Field should be the same direction of $\vec{B}$, according to the right − hand rule the I should be the direction shown above.

c. Calculate the magnitude of the current in the crossbar as it falls as a function of the crossbar's speed v.

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt}(B_0 L h) = B_0 L \frac{dh}{dt} = B_0 L v = IR$$
$$\Rightarrow I = B_0 L v / R$$

d. Derive, but do NOT solve, the differential equation that could be used to determine the speed v of the crossbar as a function of time t.

$$\sum F = mg - F_{mag} = ma \Rightarrow mg - I(L \times B) = m\frac{dv}{dt}$$
$$\Rightarrow \ mg - \frac{B_0 L v}{R}(L \times B_0) = m\frac{dv}{dt} \Rightarrow mg = \frac{B_0^2 L^2}{R} v + m\frac{dv}{dt}$$

e. Determine the terminal speed v_T of the crossbar.

$$t = \infty, mg = F_{mag} \Rightarrow mg = \frac{B_0^2 L^2}{R} v$$
$$\Rightarrow v(t = \infty) = \frac{mgR}{B_0^2 L^2}$$

f. If the resistance R of the crossbar is increased, does the terminal speed increase, decrease, or remain the same?

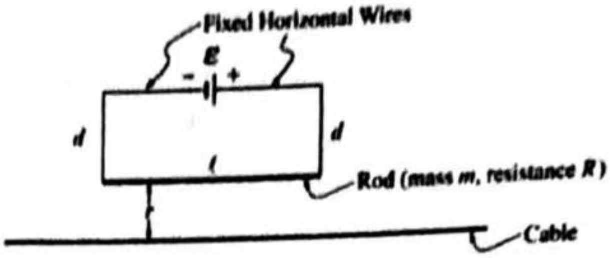
_____ Increases _____ Decreases _____ Remains the same

decrease

Give a physical justification for your answer in terms of the forces on the crossbar.

$$I = \frac{B_0 L v}{R} = \frac{B_0 L g}{R} t$$
$$\Rightarrow (B_0 L g \text{ keep same}) R \text{ increase}, I \text{ decrease}$$

7. The circuit shown consists of a battery of emf $\mathcal{E}$ in series with a rod of length ℓ, mass m, and resistance R. The rod is suspended by vertical connecting wires of length d, and the horizontal wires that connect to the battery are fixed. All these wires have negligible mass and resistance. The rod is a distance r above a conducting cable. The cable is very long and is located directly below and parallel to the rod. Earth's gravitational pull is toward the bottom of the page. Express all algebraic answers in terms of the given quantities and fundamental constants.

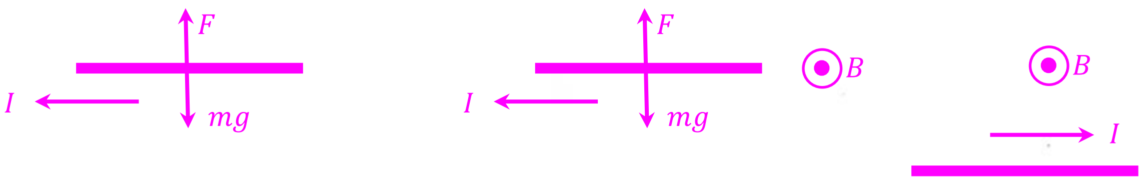


a) What is the magnitude and direction of the current I in the rod?

$$I = \frac{\mathcal{E}}{R}$$

direction: to the left

b) In which direction must there be a current in the cable to exert an upward force on the rod? Justify your answer.



c) With the proper current in the cable, the rod can be lifted up such that there is no tension in the connecting wires. Determine the minimum current I_c in the cable that satisfies this situation.

$$\text{the rod: } F = mg = I_{rod}\vec{l} \times \vec{B}$$
$$\text{the } \vec{B} \text{ by the cable: } B(2\pi r) = \mu_0 I_{cable}$$
$$\Rightarrow mg = I_{rod}\, l\left(\frac{\mu_0 I_{cable}}{2\pi r}\right) \Rightarrow I_{cable} = \frac{I_{rod}\, l 2\pi r}{\mu_0} = \frac{\mathcal{E} l 2\pi r}{R\mu_0}$$

d) Determine the magnitude of the magnetic flux through the circuit due to the minimum current I_c determined in part (c).

$$\Phi_m = \vec{B} \cdot \vec{A} = BA \xrightarrow{dA = ldr} \Phi = Bldr$$
$$B(2\pi r) = \mu_0 I_c \Rightarrow \Phi = \left(\frac{\mu_0 I_c}{2\pi r}\right) ldr = \frac{\mu_0 l \mathcal{E}}{2\pi R} \ln\frac{l + d}{l}$$